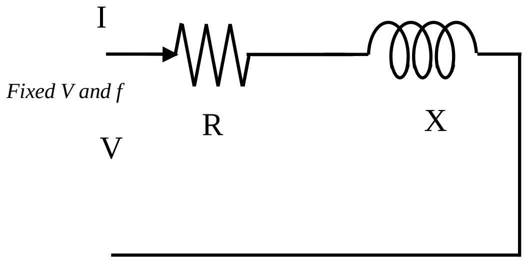
Circle diagram of Induction machine

Consider a series circuit containing reactance and a resistance connected across a fixed voltage source.

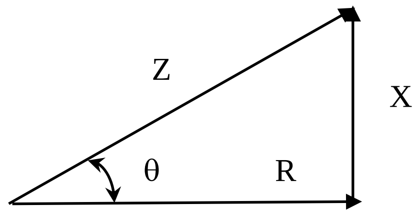


$$\vec{I} = \frac{\vec{V}}{Z} = \frac{\vec{V}}{R + jX}$$

Magnitude of Current $I = \frac{V}{\sqrt{R^2+X^2}}$

Phase angle $\theta = \tan^{-1}\frac{X}{R}$

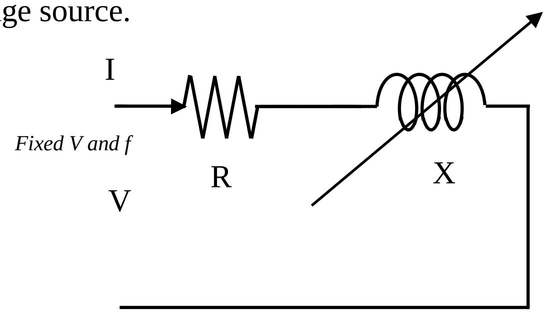
From the impedance triangle



$$\cos\theta = \frac{R}{Z};\ \sin\theta = \frac{X}{Z}; and\ \tan\theta = \frac{X}{R}$$

Case 1: Fixed Resistance and Variable reactance

Consider a series circuit containing variable reactance and a resistance connected across a fixed voltage source.



$$\vec{I} = \frac{\vec{V}}{Z} = \frac{\vec{V}}{R + jX}$$

Magnitude of Current $I = \frac{V}{\sqrt{R^2+X^2}}$

Phase angle $\theta = \tan^{-1}\frac{X}{R}$

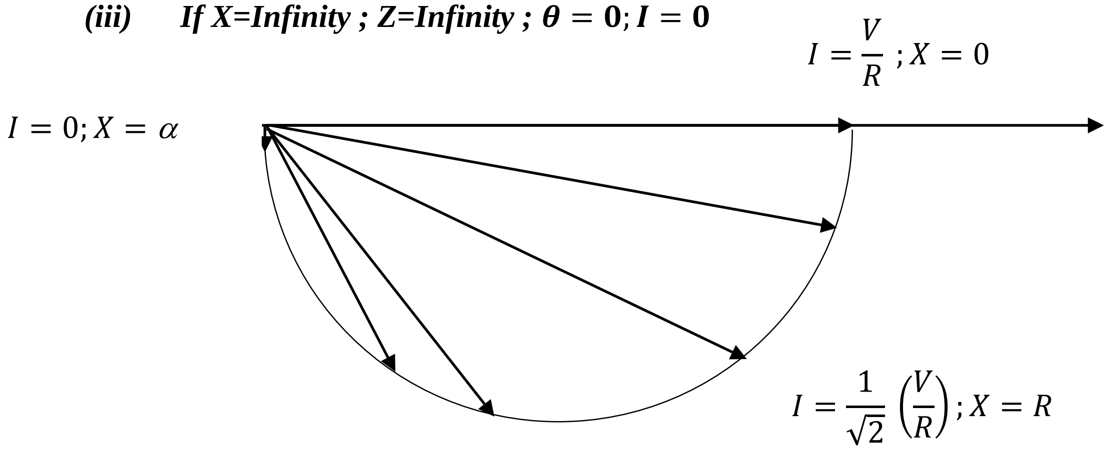
$$\mathrm{I} = \frac{\mathrm{V}}{\sqrt{\mathrm{R}^2+\mathrm{X}^2}} = \frac{\mathrm{V}}{\mathrm{Z}} = \frac{\mathrm{V}}{\mathrm{Z}} \times \frac{\mathrm{R}}{\mathrm{R}} = \left(\frac{\mathrm{V}}{\mathrm{R}}\right)\left(\frac{\mathrm{R}}{\mathrm{Z}}\right) = \left(\frac{\mathrm{V}}{\mathrm{R}}\right)\cos\theta; \text{where } \theta = \tan^{-1}\frac{\mathrm{X}}{\mathrm{R}}$$

$$\boldsymbol{I = \left(\frac{V}{R}\right)\cos\theta; where\ \theta = tan^{-1}\frac{X}{R}}$$

(i) ***If X=0; Z=R;*** $\boldsymbol{\theta = 0}$; $\boldsymbol{I = \left(\frac{V}{R}\right)}$

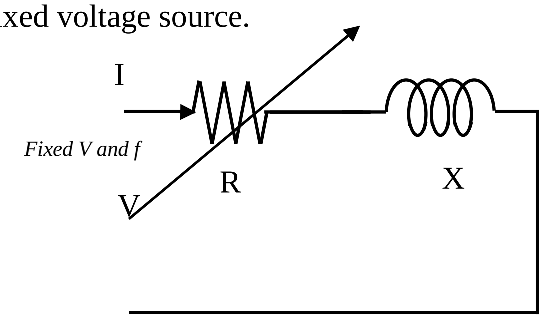
(ii) ***If X=R; Z=***$\boldsymbol{\sqrt{2}\,R}$; $\boldsymbol{\theta = 45\ degrees; I = \frac{1}{\sqrt{2}}\left(\frac{V}{R}\right)}$

(iii) ***If X=Infinity ; Z=Infinity ;*** $\boldsymbol{\theta = 0; I = 0}$



Case 2: Fixed Reactance and Variable resistance

Consider a series circuit containing variable resistance and a fixed reactance connected across a fixed voltage source.



$$\vec{I} = \frac{\vec{V}}{Z} = \frac{\vec{V}}{R + jX}$$

Magnitude of Current $I = \frac{V}{\sqrt{R^2+X^2}}$

Phase angle $\theta = \tan^{-1}\frac{X}{R}$

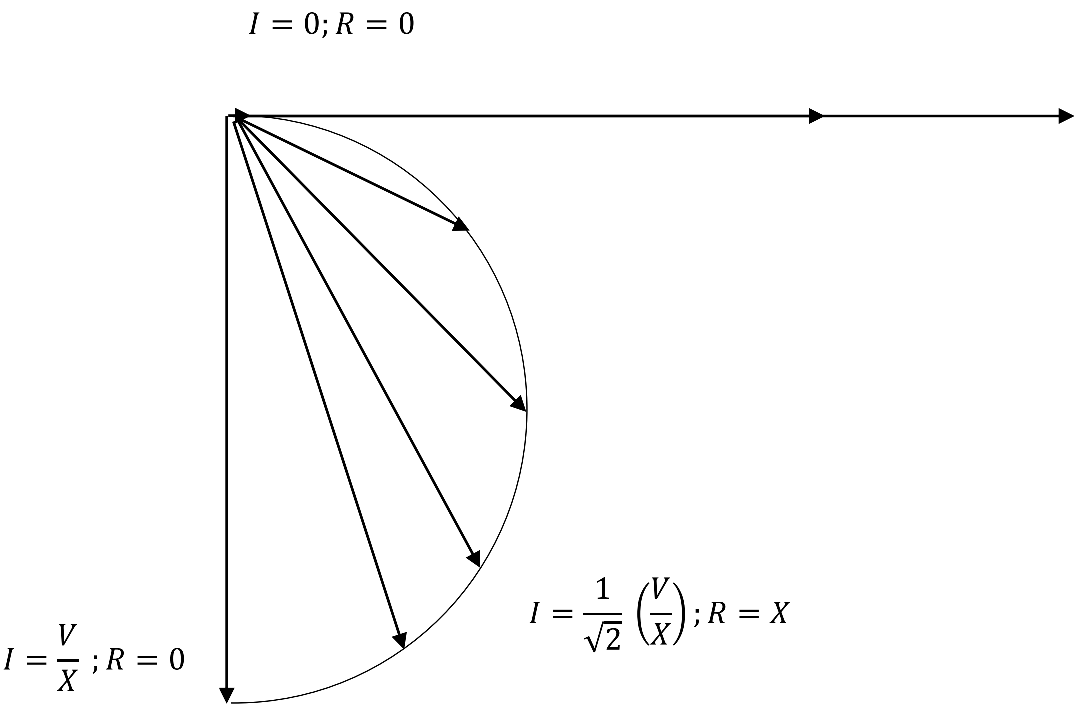
$$\mathrm{I} = \frac{\mathrm{V}}{\sqrt{\mathrm{R}^2+\mathrm{X}^2}} = \frac{\mathrm{V}}{\mathrm{Z}} = \frac{\mathrm{V}}{\mathrm{Z}} \times \frac{\mathrm{X}}{\mathrm{X}} = \left(\frac{\mathrm{V}}{\mathrm{X}}\right)\left(\frac{\mathrm{X}}{\mathrm{Z}}\right) = \left(\frac{\mathrm{V}}{\mathrm{X}}\right)\sin\theta; \text{where } \theta = \tan^{-1}\frac{\mathrm{X}}{\mathrm{R}}$$

$$\boldsymbol{I = \left(\frac{V}{X}\right)\sin\theta; where\ \theta = tan^{-1}\frac{X}{R}}$$

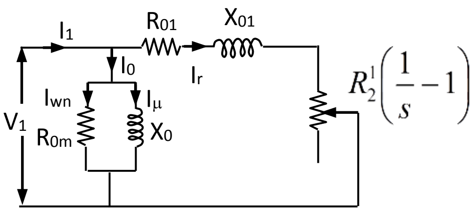
(i) ***If R=0; Z=X;*** $\boldsymbol{\theta = 90^0}$; $\boldsymbol{I = \left(\frac{V}{X}\right)}$

(ii) ***If R=X; Z=***$\boldsymbol{\sqrt{2}\,X}$; $\boldsymbol{\theta = 45\ degrees; I = \frac{1}{\sqrt{2}}\left(\frac{V}{X}\right)}$

(iii) If R=Infinity ; Z=Infinity ; $\theta = 90\ degrees; I = 0$



Consider approximate electrical equivalent circuit of Induction motor



Circle diagram of 3-ϕ induction machine is basically a current locus diagram, which represents different load conditions on shaft of the induction machine in all modes of operations.

This circuit has one fixed branch (no-load brand) and one variable branch (Load branch).

$R_2^1\left(\frac{1}{s}-1\right)$ varies from 0 to ∞.

Net resistance $R_{net} = R_{01} + R_2^1\left(\frac{1}{s}-1\right)$

Under No load condition:

Slip s = 0, rotor speed $N_r \approx N_s$ and $R_2^1\left(\frac{1}{s}-1\right) = \infty$

$\Rightarrow$ net resistance $R_{net} = \infty$

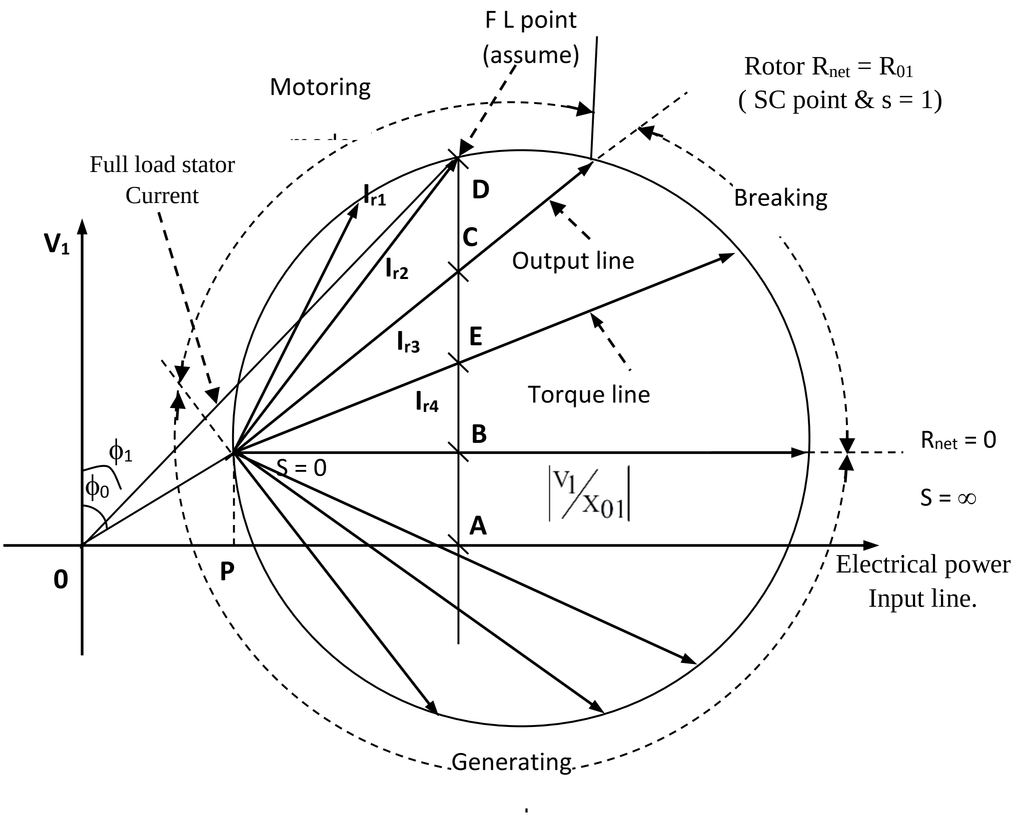
Therefore no-load current I_0 only will pass in the circuit.

Under Loaded condition:

$$\text{Slip } s\uparrow,\ \text{rotor speed } N_r\downarrow \text{ and } R_2^1\left(\frac{1}{s}-1\right)\downarrow$$

$\Rightarrow$ net resistance $R_{net} = \downarrow$ then I_2^1 current will flows

$$I_2^1 = \frac{V_1}{\sqrt{R_{net}^2 + X_{01}^2}}$$



OP = Reactive power consumed by **fixed** branch.

PA = Reactive power consumed by **load** branch.

AB = Active power consumed by **fixed** branch.

BD = Active power consumed by **load** branch.

CD = Net mechanical power output.

Observations from circle diagram:

Motoring mode ($0 < s \leq 1$)

$R_{net} = +\infty$ to R_{01}

The power will sink.

Breaking mode (1< s ≤ ∞)

R_{net} = R_{01} to 0

The power will sink.

Generating mode: (–∞ <s<0)

R_{net} = 0 to –∞ (–Ve)

The power is source of power.

If load is vary, then I_2^1 current is also changes along load.

If $T_L > T_{max}$ then operation becomes unstable (i.e rotor comes into blocked rotor condition, high current is passing)

Under blocked rotor test: N = 0, s = 1 and $R_2^1\left(\frac{1}{s}-1\right)=0$

$$R_{net} = R_{01} + R_2^1\left(\frac{1}{s}-1\right)$$

$$\therefore\ I_r = \frac{V_1}{\sqrt{\left(R_{01}^2 + X_{01}^2\right)}} \qquad \text{and} \quad \phi = \tan^{-1}\left(\frac{X_{01}}{R_{01}}\right)$$

- (reduced voltage V_{BR}) $\rightarrow I_{BR} = I_2$ (*Blocked rotor current*)

  (Rated voltage V_{rated} $\rightarrow$ (Short circuit current I_{SC} = ?)

  $$I_{SC} = I_{BR} \times \frac{V_{rated}}{V_{BR}}$$

- Breaking mode of operation (S>1)

  More +ve   − ve

  $$R_{net} = R_{01} + R_2{}^1\left(\frac{1}{S}-1\right)$$

  $$= R_2^1 + R_2{}^1\left(\frac{1}{S}-1\right) \qquad \because R_{01} = R_1 + R_2^1 \text{ and } R_{01} \cong R_2^1$$

  $$= +Ve$$

  Therefore I_2 is flows (Still machine consumes power in braking mode)

- In braking mode, the electrical power supplied to the mains is enough to meet the losses in the motor therefore generated power to the supply system is Zero.

At end of Breaking, s=∞.

$$R_{net} = R_{01} + R_2{}^1\left(\frac{1}{s}-1\right) \Rightarrow 0$$

$$\therefore I_r{}^1 = \frac{V_1}{jx_{01}} \angle -90^o$$

In this position, machine neither generates power to back nor supply draw from the source.

Generator mode:

N>Ns and S = –Ve, then $R_2^1(\frac{1}{s}-1) \Rightarrow -Ve$

$R_{net} \approx R_2^1 + R_2^1(\frac{1}{s}-1)$

+Ve < –Ve

Ex:- power drawn by R_2^1 is 100W and power generated by $R_2^1\left(\frac{1}{s}-1\right)$ is 200W then net out put electrical power = 100W.

Construction of the Circle Diagram

The performance characteristics of an induction motor derived from a circular locus are discussed. The required data to draw the circle diagram may be found from the no load test and blocked rotor tests, corresponding to the open-circuit and short circuit tests of a transformer. The stator and rotor copper losses can be separated by drawing a torque line. The parameter, of the motor in the equivalent circuit can be found from the tests discussed.

Data Required

From No-load

(i) No-load Current Io

(ii) No-load power factor $\cos\Phi_o$

From Blocked Rotor test

(i) Short Circuit power factor $\cos\Phi_{sc}$

(ii) Short Circuit Current I_{sc}, from this calculate ISN, which would be the current that would flow in the motor winding at blocked rotor conditions with rated voltage using the following relations

$$I_{SN} = \left(\frac{V_{rated}}{V_{sc}}\right) \times I_{sc}$$

(iii) Power consumed at blocked rotor test, from that, W_{SN}, where

$$W_{SN} = \left(\frac{V_{rated}}{V_{sc}}\right)^2 \times W_{sc}$$

1. Take reference phasor V as Vertical (Y-axis)
2. Select suitable current scale such that diameter of the circle is about 20 to 30 cm
3. From no load test, I_o and $\emptyset_0$ are obtained. Draw Vector I_o , lagging V by angle $\emptyset_0$. This is line OA as shown in the figure
4. Draw horizontal line through extremity of I_o i.e A , parallel to horizontal axis
5. Draw the current I_{SN} calculated from I_{sc} with the same scale, lagging V by angle $\emptyset_{sc}$ from the origin O. This is phase OC as shown in the figure
6. Join AC , the line AC is called ***output line***
7. Draw a perpendicular bisector of AC , extend it to meet line AB at point D

8. Draw the circle, with D as a centre and radius equal to AD. This meets the horizontal line drawn from A at B as shown in the figure
9. Draw the perpendicular from point C on the horizontal axis , to meet AB line at G and meet horizontal axis at E
10. Torque Line: The torque line separates stator and rotor copper losses
    Note: Voltage axis is vertical, all the vertical distances are proportional to active components of currents of power input, if measured appropriate scale

The vertical distance CE represents power input at short circuit i.e W_{SN}, which consists of core loss and stator, rotor copper loss

GE= AA'=Fixed losses

Where AA' is drawn perpendicular from A on horizontal axis. This represents power input on no load i.e fixed losses

Hence CG proportional to stator and rotor copper losses

Then point F can be located as

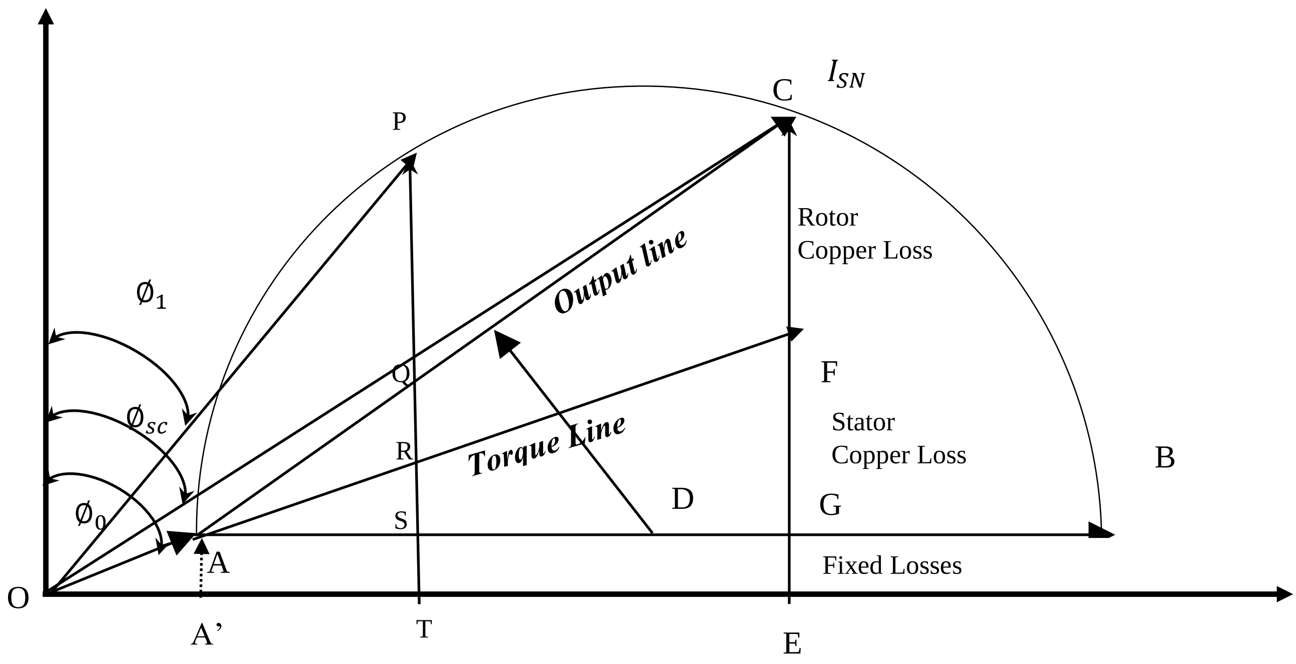
$$\frac{CF}{FG} = \frac{rotor\ copper\ loss}{stator\ copper\ loss}$$

The line AF under this condition is called ***torque line***

POWER SCALE: As CE represent W_{SN} i.e power input on short circuit at normal voltage, the power scale can be obtained as

$$Power\ Scale = \frac{W_{SN}}{l(CE)}\ W/cm$$

Where l(CE) = Distance CE in cm



Location of Point F: In a slip ring Induction motor, the stator resistance per phase R_1 and rotor resistance per phase R_2 can be easily measured. Similarly by introducing ammeters in stator and rotor circuits, the currents I_1 and I_2 also can be measure

$$K = \frac{I_1}{I_2} = transformation\, ration$$

$$\frac{CF}{FG} = \frac{rotor\ copper\ loss}{stator\ copper\ loss} = \frac{I_2^2 R_2}{I_1^2 R_1} = \frac{R_2}{R_1}\left(\frac{I_2}{I_1}\right)^2 = \frac{R_2}{R_1}\frac{1}{K^2}$$

$$R_2' = \frac{R_2}{K^2} = rotor\ resistance\ referred\ to\ stator\ side$$

$$\frac{CF}{FG} = \frac{R_2^1}{R_1}$$

Thus point F be obtained by dividing line CG in the ratio of R_2' to R_1

In a squirrel cage motor, the stator resistance can be measured by conduction resistance test

Stator copper loss=$3I^2_{SN}R_1$, where I_{SN} is phase value

Neglecting core loss, WSN= Stator copper loss +Rotor Copper loss

Therefore Rotor Copper loss=W_{SN}-$3I^2_{SN}R_1$

$$\frac{CF}{FG} = \frac{W_{SN} - 3I_{SN}^2 R_1}{3I_{SN}^2 R_1}$$

Dividing the line CG in this ration, the point F can be obtained and hence AF represents torque line

Predicting performance from Circle diagram

Let motor is running by taking a current OP as shown in the figure. The various performance parameters can be obtained from the circle diagram at that load condition

Draw perpendicular from point P to meet output line at Q, torque line at R, the base line at S and horizontal axis at T.

We know the power scale as obtained earlier.

Using the power scale and various distances, the values of the performance parameters can be obtained as,

1. Total motor input = PT× Power Scale
2. Fixed loss = ST× Power Scale
3. Stator Copper loss = SR× Power Scale
4. Rotor Copper loss = QR× Power Scale
5. Total loss = QT× Power Scale
6. Rotor output = PQ× Power Scale
7. Rotor input = PQ× QR= PR× Power Scale
8. Slip $s = \frac{rotor\ copper\ loss}{rotor\ input} = \frac{QR}{PR}$
9. Power factor $\cos\emptyset = \frac{PT}{OP}$
10. Motor efficiency =$\frac{Output}{input}$ =$\frac{PQ}{PT}$
11. Rotor efficiency =$\frac{Rotor\ output}{Rotor\ input}$ =$\frac{PQ}{PR}$
12. $\frac{Rotor\ output}{Rotor\ input}$ =1-s =$\frac{N_r}{N_s}$ =$\frac{PQ}{PR}$

The torque is the rotor input in synchronous watts.

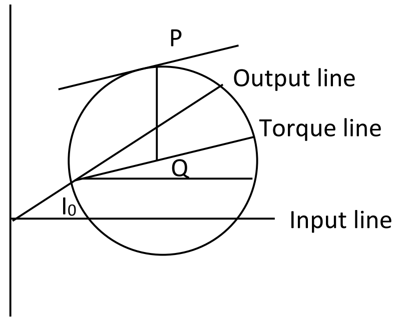
Maximum Quantities

The maximum values of various parameters can also be obtained by using circle diagram.

1. **Maximum Torque**: Draw a line parallel to AF(torque line) and is also tangent to the circle at point P. The point P can also obtained by extending the perpendicular drawn from D on AF to meet the circle at P. Then the maximum output is given by *l*(PQ) at the power scale. This is shown in the fig.

PQ = Max Rotor Input,

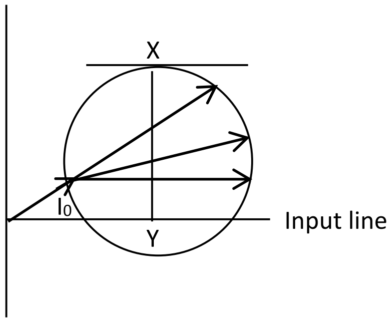
Maximum torque $T_{\max} = \frac{60}{2\pi Ns} \times PQxPowerscale$



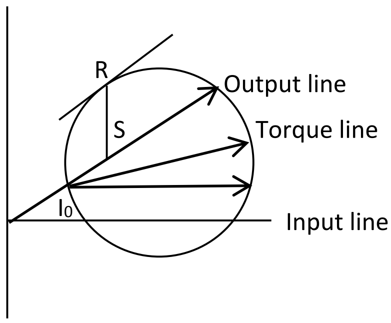
2. **Maximum Input** : It occurs at the highest point on the circle i.e. at point X. At this point, tangent to the circle is horizontal .The maximum input is given *l*(XY) at the power scale.

XY = maximum electrical power input

P_{inmax}=l(XY)*x Power Scale*



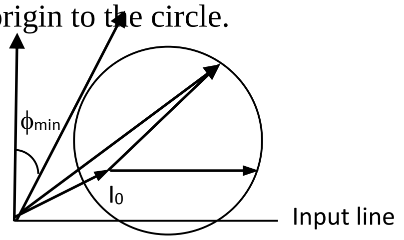
3. **Maximum Output** : Draw a line parallel to the output line and is also tangent to the circle at point R. The point R can also be obtained by drawing perpendicular from D on torque line and extending it to meet circle at point R. The *l*(RS) represents maximum torque in synchronous watts at the power scale. This torque is also called **stalling torque or pull out torque**.



Maximum output power=l(RS)x Power Scale

4. **Maximum Power Factor**: Draw tangential from origin to the circle.

Maximum power factor = Cosϕ_{min}.



5. **Starting Torque** : The torque is proportional to the rotor input .At s =1,rotor input is equal to rotor copper loss i.e.l(CG).

$\therefore T_{start}$ = l(CG)× power scale ………in synchronous watts

❖ **Full Load Condition**

The full Load Motor output is given on the name plates in watts or h.p. calculate the distance corresponding to the full load output using the power scale.

Then extend CG upwards from A onwards, equal to the distance corresponding to full load output, say C' draw parallel to the output line AC from C' to meet the circle at point P'. This is the point corresponding to the full load condition, as shown in the fig.

Once point P' is known, the other performance parameters can be obtained easily as discussed above.